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The proper treatment of linguistic ambiguity in ordinary algebra

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Formal Grammar, Bozen, 20.08.2016

SFB 991

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Overview

(1) *bank* ('financial institute', 'strip of land along a river')

Old puzzle:

- If we treat ambuigity as a semantic phenomenon (i.e. assume the existence of *ambiguous meanings*),
- the question arises: what is the meaning of ambiguity?
- In particular, one has to account for the property of *universal distribution* (see below)

Overview

Answers:

- We provide a simple axiomatization which captures all combinatorial aspects of ambiguity in the context of Boolean algebras.
- But: from these axioms (which are correct beyond doubt) a lot of properties follow which are *not appropriate* for ambiguity.

New puzzle:

- If the most obvious properties of ambuity lead to properties which are obviously wrong – how do we get out of this?
- One possibility: ambiguity is not *total*, rather a partial operator.
- Another possibility: natural language semantics is not Boolean in nature.

Outline

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Linguistic ambiguity

Linguistic ambiguity

Exponents of natural language give rise to two or more sharply distinguished meanings.

(2) a. *bank* ('financial institute', 'strip of land along a river')
b. *kick the bucket* ('kick the bucket', 'die')
c. *see the man with the telescope* ('see with', 'man with')
d. *every boy loves a movie* ($\forall\exists$, $\exists\forall$)

Syntactic versus semantic approach

Say:

e	is an EXPONENT	(*bank*)
m_1	is one MEANING of e	('financial institute')
m_2	is another MEANING of e	('river bank')
(e, m)	is a SYMBOL	

Syntactic approach

Two separate symbols (e, m_1) and (e, m_2).

Semantic approach

One ambiguous symbol $(e, m_1 \parallel m_2)$.

Question: What does $\parallel$ actually mean?

Advantages of the semantic approach

Semantic approach

One ambiguous symbol ($e, m_1 \parallel m_2$).

Advantages:

- function from form to meaning
- more succinct lexicon
- interaction of ambiguity, meaning composition, and inference

(3) a. *The federal agency decided to take the project under its well-muscled, federally-funded wing.*[1]
b. *We pulled his cross-gartered leg.*[1]

(4) *The first thing that strikes a stranger in New York is a big car.*[3]

What || doesn't mean

Question: What does || actually mean?

Quick shot: Disjunction!

$$m_1 \parallel m_2 \equiv m_1 \vee m_2$$

Hence: $\neg(m_1 \parallel m_2) \equiv \neg(m_1 \vee m_2) \equiv \neg m_1 \wedge \neg m_2$

Yet intuitively: $\neg(m_1 \parallel m_2) \equiv \neg m_1 \parallel \neg m_2$

(5)#*There is no bank.*

'There is no financial institute and there is no strip of land along the river.'

(see also Pinkal [2], Poesio [3], and Stallard [4])

Outline

The semantics of linguistic ambiguity

We investigate the meaning of || in the context of **Boolean algebras**.

- This is an important restriction: Boolean algebras correspond (via algebraization) to classical logic – which might not necessarily the right logic for NL-semantics.
- Still, the vast majority of semanticists interpret connectives in classical logic – so for all choices, this seems to be the most natural!
- In the algebraic approach, algebraic $\leq$ (defined by $a \leq b \Leftrightarrow a \wedge b = b$) corresponds to logical $\vdash$ and semantic $\models$

The semantics of linguistic ambiguity

Question: What are the semantic properties of $\|$?

In terms of **denotation**, there is a fundamental difference between disjunction and ambiguity: the meaning of an ambiguous statement depends on the underlying (often unknown) **intention** of the speaker:

Intentionality

$$a \parallel b \leq a \vee b \tag{6}$$

$$a \leq a \vee b \tag{7}$$

$$\textbf{But: } a \nleq a \parallel b \tag{8}$$

I need some money! $\nleq$ *I need some dough!*
I need some pastry or some money! $\neq$ *I need some dough!*

The semantics of linguistic ambiguity

The **combinatorial properties** of ambiguity are different from disjunction: ambiguity has the property of universal distribution:

Universal distribution

$$\sim(a \parallel b) = \sim a \parallel \sim b \tag{9}$$

$$(a \parallel b) \vee c) = (a \vee c) \parallel (b \vee c) \tag{10}$$

$$(a \parallel b) \wedge c) = (a \wedge c) \parallel (b \wedge c) \tag{11}$$

$$(a \parallel b) \rightarrow c = (a \rightarrow c) \parallel (b \rightarrow c) \tag{12}$$

$$a \rightarrow (b \parallel c) = (a \rightarrow b) \parallel (a \rightarrow c) \tag{13}$$

The semantics of linguistic ambiguity

There are some additional properties of $\|$:

Associativity

$$(a \parallel b) \parallel c = a \parallel (b \parallel c) \tag{ass}$$

Idempotence

$$a \parallel a = a \tag{id}$$

Commutativity (arguable)

$$a \parallel b = b \parallel a \tag{com}$$

Outline

Uniform Usage

The following hypothesis of uniform usage is necessary for an algebraic treatment of ambiguity:

Uniform usage (UU)

In a given context, an ambiguous statement is used consistently in **only one** sense.

This leaves of course many things unspecified (as context), but allows us to treat || as an algebraic operator (which is a function!)

Ambiguous algebras

Ambiguous algebra

An AMBIGUOUS ALGEBRA is a structure $\mathbf{A} = (A, \wedge, \vee, \sim, \|, 0, 1)$, where $(A, \wedge, \vee, \sim, 0, 1)$ is a Boolean algebra, and $\|$ is a binary operation for which the following holds:

$$\sim(a \parallel b) = \sim a \parallel \sim b \tag{$\parallel$ 1}$$

$$a \wedge (b \parallel c) = (a \wedge b) \parallel (a \wedge c) \tag{$\parallel$ 2}$$

$$\text{At least one of} \quad a \leq a \parallel b \quad \text{or} \quad b \leq a \parallel b \quad \text{holds} \tag{$\parallel$ 3}$$

Ambiguous algebras: a peculiar axiom

$$\text{At least one of} \quad a \leq a \parallel b \quad \text{or} \quad b \leq a \parallel b \quad \text{holds} \qquad (\parallel 3)$$

Note that ($\parallel$ 3) is a **disjunction**!

- This entails, among other, that there is no *free ambiguous algebra*, a central tool in general algebra.
- Put differently, in *every* ambiguous algebra some equalities hold which do not hold in all ambiguous algebras (this nicely models the epistemic aspect of ambiguity)
- To the best of our knowledge, axioms of this kind have not been considered in general algebra so far. Any algebraist know better?

Questions regarding the axiomatization

1. Do these axioms entail all properties we find intuitively true for ambiguity?
   – As far as we can see, clearly yes.

2. Do they imply some properties we find intuitively incorrect for ambiguity in general?
   – Unfortunately, also clearly yes.

3. Do non-trivial algebras exist which satisfy these axioms? (That is, for example, algebras with more than one element?)
   – Clearly yes, but if we add commutativity for $\|$, then no.

4. Are there ambiguous algebras, where $a \parallel b \neq a$ and $a \parallel b \neq b$?
   – No, there are not.

Ambiguous algebras: An example

Take the obvious Boolean algebra over the set $\{0, a, b, 1\}$. Put

$$\begin{array}{llll} a \parallel b = a; & b \parallel a = b; & 0 \parallel a = 0; & 1 \parallel a = 1; \\ a \parallel 1 = a; & b \parallel 1 = b; & 0 \parallel b = 0; & 1 \parallel b = 1; \\ a \parallel 0 = a; & b \parallel 0 = b; & 0 \parallel 1 = 0; & 1 \parallel 0 = 1; \end{array}$$

$\wedge$-distribution holds:

$$a = a \parallel b = (a \parallel b) \wedge a = a \parallel 0 = a$$

$$0 = 0 \parallel 1 = (0 \parallel 1) \wedge a = 0 \parallel a = 0$$

and so on, same for $\vee, \sim$

We thus have a proper non-trivial 4-element algebra.

Outline

Structure theory I: Uniformity

Lemma 1

In every ambiguous algebra $\mathbf{A}$ and for all $a, b \in A$,
either $\quad a = a \parallel b \quad$ or $\quad b = a \parallel b$.

No commutativity for $\parallel$:

assume	$a \parallel {\sim}a = a$
then	${\sim}a \parallel a = a$
but also	${\sim}({\sim}a \parallel a) = {\sim}{\sim}a \parallel {\sim}a = a \parallel {\sim}a = a$
hence	${\sim}a = a$ (which only holds in 1-element algebras)

Linguistically relevant?

(14) *sacré* ('cursed', 'holy')

Structure theory I: Uniformity

Lemma 1

In every ambiguous algebra $\mathbf{A}$ and for all $a, b \in A$,
either $\quad a = a \parallel b \quad$ or $\quad b = a \parallel b$.

Corollary 2

If $\mathbf{A}$ is an ambiguous algebra such that for all $a, b \in A$, $a \parallel b = b \parallel a$, then A has at most one element.

Corollary 3

For all ambiguous algebras $\mathbf{A}$, $a, b \in A$, we have

1. $a \parallel a = a$
2. $a \wedge b \leq a \parallel b \leq a \vee b$

Structure theory I: Uniformity

Lemma 4 (Monotonicity of $\|$)

Assume $a' \leq a$ and $b' \leq b$. Then $a' \parallel b' \leq a \parallel b$.

Lemma 7 (Uniformity lemma)

Assume we have an ambiguous algebra $\mathbf{A}$ $a, b \in A$ such that $a \neq b$.

1. If $a \parallel b = a$, then for all $c, c' \in A$, we have $c \parallel c' = c$;
2. if $a \parallel b = b$, then for all $c, c' \in A$, we have $c \parallel c' = c'$.

Corollary 8

If $\mathbf{A}$ is an ambiguous algebra, $a, b \in \mathbf{A}$, then either for all $a, b \in A$, we have $a \parallel b = a$, or for all $a, b \in A$, we have $a \parallel b = b$.

Hence: Ambiguous algebras are either RIGHT-SIDED or LEFT-SIDED.

Outline

The meaning of monotonicity

Does monotonicity go together with intuitions about ambiguity?

Lemma 4 (Monotonicity of $\|$)

Assume $a \leq a'$ and $b \leq b'$. Then $a \,\|\, b \leq a' \,\|\, b'$.

$\Longrightarrow$ This roughly means: ambiguity respects logical entailment.

Example 1:

bank	means	$a \,\|\, b$
restaurant	means	c
bank or restaurant	means	$(a \,\|\, b) \vee c = (a \vee c) \,\|\, (b \vee c) = a' \,\|\, b'$

$\Longrightarrow$ If monotonicity were wrong, then *bank* would not (generally) entail *bank or restaurant*!

The meaning of monotonicity

Does monotonicity go together with intuitions about ambiguity?

Lemma 4 (Monotonicity of $\|$)

Assume $a \leq a'$ and $b \leq b'$. Then $a \,\|\, b \leq a' \,\|\, b'$.

$\Longrightarrow$ This roughly means: ambiguity respects logical entailment.

Example 2:

bank	means	$a \,\|\, b = a$
bank or restaurant	means	$(a \,\|\, b) \vee c = (a \vee c) \,\|\, (b \vee c) = a' \,\|\, b'$
kank	means	$a' \,\|\, b' = a \vee c$

$\Longrightarrow$ Hence monotonicity is like uniform usage for expressions which are connected by the relation of entailment.

The meaning of uniformity

Does uniformity go together with intuitions about ambiguity?

Lemma 7 (Uniformity lemma, informal)

Ambiguous algebras are **either** left-sided **or** right-sided.

Hence: $a \parallel b = a$ **or** $a \parallel b = b$ for any a, b

Example: *bank* means 'financial institute', and **accordingly** *dough* means 'pastry'.

This is completely unintuitive, and a consequence of two things:

1. the strong axioms of Boolean algebras, in particular the equality $\sim\sim a = a$, and
2. the fact that $\parallel$ is a total operator (i.e., for all a, b, we have an object $a \parallel b$).

Ways out of the dilemma

As there can be hardly any doubt about (||1–3), there are only two ways out:

1. Boolean algebras/classical connectives are inadequate for NL semantics. Most of the results presented here do no longer hold with Heyting algebras/intuitionistic logic.
2. Ambiguity is a **partial operator**, and partially ambiguous algebras are the smallest Boolean algebras generated by some ambiguous terms. In particular, the uniformity lemma depends on the existence of the object $0 \parallel 1$, which has no linguistic motivation and need not exist in partial algebras.

Outline

Structure theory II: Completions

So far, we have exposed restrictions on ambiguous algebras.

In this part, we show results on the existence of (non-trivial) ambiguous algebras.

Structure theory II: Completions

Let `id` be the identity map. Every Boolean algebra can be completed to an ambiguous algebra without collapsing elements:

Lemma 17 (Completion lemma)

Every Boolean algebra $\mathbf{B} = (B, \wedge, \vee, \sim, 0, 1)$ can be completed to an ambiguous algebra $\mathbf{A} = (B, \wedge, \vee, \sim, \|, 0, 1)$ such that the map $\mathtt{id} : \mathbf{A} \to \mathbf{B}$ is a Boolean algebra isomorphism.

Structure theory II: Completions

Together with the uniformity lemma, this proves the following:

Corollary 18

For every Boolean algebra $\mathbf{B} = (B, \wedge, \vee, \sim, 0, 1)$, there are exactly two ambiguous algebras $\mathbf{A}$ such that the map $\mathtt{id} : \mathbf{A} \to \mathbf{B}$ is a Boolean algebra isomorphism.

That is: for every Boolean algebra there is one left-sided and one right-sided completion.

Decidability and complexity

This means that decidability results for Boolean algebras can be transferred to ambiguous algebras.

- every AA-term t, knowing it is interpreted in a left-(right-)sided ambiguous algebra, can be reduced to a Boolean algebra term.
- To check whether $t_1 = t_2$ holds in all ambiguous algebras, just check:
  1. for t_1, t_2 terms in a left-sided algebra, for their Boolean reductions $b_l(t_1), b_l(t_2)$, $b_l(t_1) = b_l(t_2)$ holds in all Boolean algebras (NP complete).
  2. same for right-sided algebras.
- The equation holds in both previous cases if and only if it holds in all ambiguous algebras.

Decidability and complexity

So what does this mean?

- We see that ambiguos Boolean algebras are quite well-behaved,
- but on the downside, we also see that there is little of interest to say about them, as they are very similar to Boolean algebras.
- This in itself is however a non-trivial result, which probably does not extend to other (weaker) types of algebras.

To cite the paper: "Still we consider it important to have established these results, which are really [not] obvious or trivial."

Outline

Conclusion and further work

Ambiguous algebra

Boolean algebra + || operator and axiomatization

Does it match linguistic intuition?

⇒ **monotonicity lemma:** words in entailment relations are used consistently meaningwise. ✓

⇒ **uniformity lemma:** ambiguous terms are either right- or left-ambiguous. ↯

How to make it fit better?

- more general class of algebras? (e.g., Heyting algebras or distributive, modular, or residuated lattices)
- partial ||?

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